Differential equations of a U-Ra-Pb system accounting for a diffusion of components.

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Decay series are considered in textbooks on isotopy. Here we bring solutions of equations for a process dissipated in space, which is characteristic for contamination.. As to our knowledge, the differential equations below are original and were never published.

If the isotope of the first substance has a concentration N_1 and decays into an isotope of the second substance with a concentration N_2 and the second substance decays finally into a third substance with a concentration N_3 assuming a diffusion of components as possible then a one-dimensional model would be completely determined by a system of differential equations

$$\frac{\partial N_1}{\partial t} = D_1 \frac{\partial^2 N_1}{\partial x^2} - \lambda_1 N_1$$

$$\frac{\partial N_2}{\partial t} = D_2 \frac{\partial^2 N_2}{\partial x^2} - \lambda_2 N_2 + \lambda_1 N_1 \qquad (1)$$

$$\frac{\partial N_3}{\partial t} = D_3 \frac{\partial^2 N_3}{\partial x^2} + \lambda_2 N$$

This situation is realized in a system **U-Ra-Pb,** with

$$N_1 = {}^{238}U$$

$$N_2 = Ra$$

$$N_3 = {}^{206}Pb$$

$$D_1 = D_3 = 0$$

Only Ra *is* suggested as diffusing and the following differential equation becomes valid (due to a small lifetime of ***Ra*** the system fast reaches a stationary regime):

$$D_2 \frac{\partial^2 Ra}{\partial x^2} - \lambda_2 Ra + \lambda_1 {}^{238}U = 0 \qquad (2)$$

$$-h/2<x<h/2$$

Solving this equation we obtain a formula for the ***Ra***-concentration

$$Ra = \frac{\frac{-\lambda_1}{\lambda_2} {}^{238..}U}{ch \frac{h}{2}\sqrt{\frac{\lambda_{..2}}{D._2}}} chx\sqrt{\frac{\lambda_2}{D_2}} + \frac{\lambda_1}{\lambda_2} {}^{238}U \qquad (3)$$

A coefficient of Ra losses is obtained if we determine the derivative of ***Ra*** at the boundary and multiply it by

D_2

$$k = \frac{2}{h}(th \frac{h}{2}\sqrt{\frac{\lambda_2}{D_2}})\sqrt{\frac{D_2}{\lambda_2}} \qquad (4)$$

Or, in a more conveniant form

$$a = 2\sqrt{\frac{D_2}{h^2 \lambda_2}} \qquad k = ath\frac{1}{a}$$

For the Pb-concentration the following formula is obtained.

$$^{206}Pb = {}^{238}U_0 (1 - e^{\lambda_1 t})\left(1 - \frac{ch\frac{2x}{ha}}{ch\frac{1}{a}}\right) \qquad (5)$$

The total quantity of residual ***Pb*** is given by

$$Pb = U_0(1 - \exp\lambda_1 t)(1 - k) \qquad (6)$$

Where k is given by (4).

These formula can be used for estimates of dissipated unfavorable quantities of the ***U-Ra-Pb*** system..