Are faults fractals?

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Abstract

A universal scaling law would be imperative to understand the processes of the evolution of faults. Searching for these laws many different data sets have been analysed. Very often a power-law distribution is found, what indicates a fractal distribution. But in the examination of fault-sets from different geological and tectonic settings the calculated relations often deviate from this. Therefore it would be interesting to compare the results of different authors in the intention to understand the causes of the numerous differing results.

1 Introduction

The study of the scaling laws that can be derived from measuring fault length, displacement and number of faults has been an active field of research in the last 20 years.

Fractures in the earth crust often generate during earthquakes, which are very well investigated. The Gutenberg-Richter-law has been a recognized property of earthquakes for over 50 years. It concerns the distribution of earthquakes. Many forms of that law have been discussed, but in the last years a power-law form was favoured by many authors. In terms of seismic moment it reads as follows:

$$N(M) = aM^{-B}, \tag{1}$$

where M is the seismic moment and N(M) is the number of earthquakes larger than M. The exponent B is always found close to a universal value of 2/3. Scholz et al.(1993) distinguished small and large earthquakes with B changing from 2/3 to 1 from small to large ones as a consequence of the scaling law that changes at that point.

Geologists have been searching intensively for uniform distribution laws of faults because that would represent important constraints on models of fault growth. It would help towards modelling and interpreting fault development in the brittle crust.

For economical purposes it is important to understand the multiphase flow in fractured hydrocarbon reservoirs and for the siting of hazardous waste disposal in crystalline rocks, because the complex fracture pattern geometry controls the overall flow and transport regimes.

Besides some scientists want to applicate these laws on earthquake hazard assessment and they want to find a possibility of prediction.

2 Definition of fractals

It is often said in the literature, that faults have a fractal distribution. In the following it is defined what "fractal" means in this case and why this distribution is so popular.

The word was defined by MANDELBROT (1987). He used the latin word *fractus* which means breaking or making irregular fractures. He further defined: " A fractal by definition is a set, the Hausdorff- Besicovitch dimension of which exceeds the topological dimension."

The best known fractal is maybe the coast of Great Britain. No one is able to say, how long it is.The minimum length of a coastline must be the distance from its start point to its end point, measured on a straight line. But everyone will agree, that it is much longer in reality. Looking closer to it step by step, it becomes longer, because one is able to see and measure more and more details. Therefore the length of a coastline is not defined and seems to be not suitable as describtion term. Now we need another term to compare different coastlines.

RICHARDSON (1961) made experimental length measurements on different curves (Figure 1). He used polygons with increasing smaller calibration lengths ε for the measurements. This is compareable with a pair of compasses, that is adjusted on this calibration length. "Walking" along the coast with this pair of compasses, an approximated length $L(\varepsilon)$ can be nominated by multiplying the number of steps by ε . With smaller ε we expect $L(\varepsilon)$ to become stable at the real length.

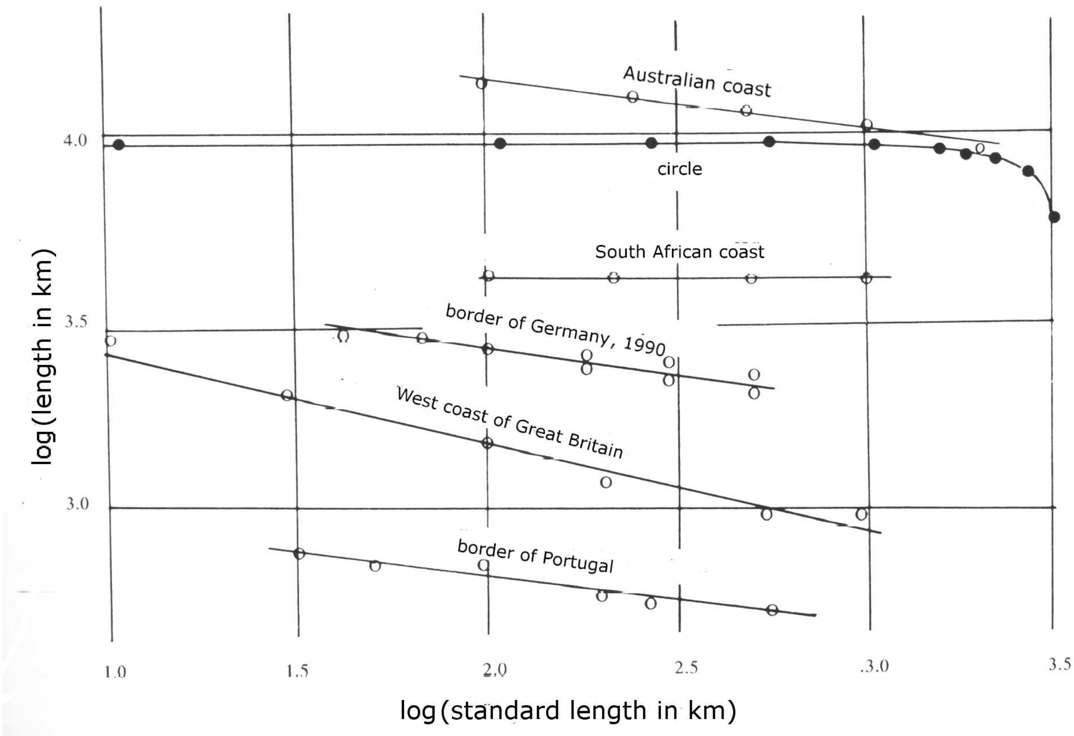

Figure 1: This diagram shows the experimental measurements of RICHARDSON(1961). He did not interpret the slopes of the curves. In MANDELBROT(1987) the graphs are seen as approximately fractal curves. The slopes are interpreted as an approximation for 1-D. (D is the fractal Dimension). (modified from MANDELBROT, 1987)

In Figure 1 it is obvious, that a circle reaches such a stable length, but coast lines and country borders never do. In this diagram, the axes of which are both logarithmic, straight lines appear. These lines follow the term

$$L(\varepsilon) = \lambda\varepsilon^{1-D} \tag{2}$$

where λ and D are constants. MANDELBROT later defined, that D is the fractal dimension. After Hausdorff D should be independent from ε. D can be a noninteger and for real fractal distributions it is larger than the known topological dimension.

Comparing equation 1 with equation 2 a convincing similarity is obvious. It seems as if the fractal nature of earthquakes has been known for 50 years, but not recognized. If earthquakes, which are so closely connected to faults are fractals, it would be not surprising, if faults are distributed the same way.

Another argument for faults being fractals is the scale invariance, which is a property of fractals. This property can also be called self-similarity. If the strain increases, the length and the displacement of a fault should always show the same proportions relative to each other. The problem in finding a fractal dimension or another distribution of the data is the relative poverty of data. Complete data sets are rarely found.

3 Methods

Some problems arise from the numerous methods to collect data. The most common way is to take the maximum displacement or the length of the fault trace of a single set of faults or even detailed profiles along fault scarps. Some authors only summarized data gathered by other scientists for further analysis. In the literature one can also find examples of clay extension experiments and of numerical modelling.

Being sure to find a fractal distribution, some authors tried to find the fractal (or Hausdorff-) dimension of a set of faults by different methods suggested by MANDELBROT (1987).

One classical definition of that dimension is the number of boxes, circles or spheres with characteristic sizes, that are necessary to cover (or include) a part of the examined object (Fig.2a). This method is called box-counting and is similar to the method with the pair of compasses (section 2), just extended in the second topological dimension, which is represented by a plane.

Figure 2b illustrates the method of the mass dimension ($D(M)$), which can also be used and should lead to the same results. Here the total fracture length lying in circles of radius r is calculated. The circles are centered on the fracture pattern. With varying r the length $L(r)$ should follow the term $L(r) = r^{D_M}$.

4 Recent results

4.1 Experimental data and numerical modelling by SPYROPOULOS et al. (2002)

This letter appears to be representative for the most recent conclusions drawn on the distributions of faults. The motivation for the work of these authors was, that besides

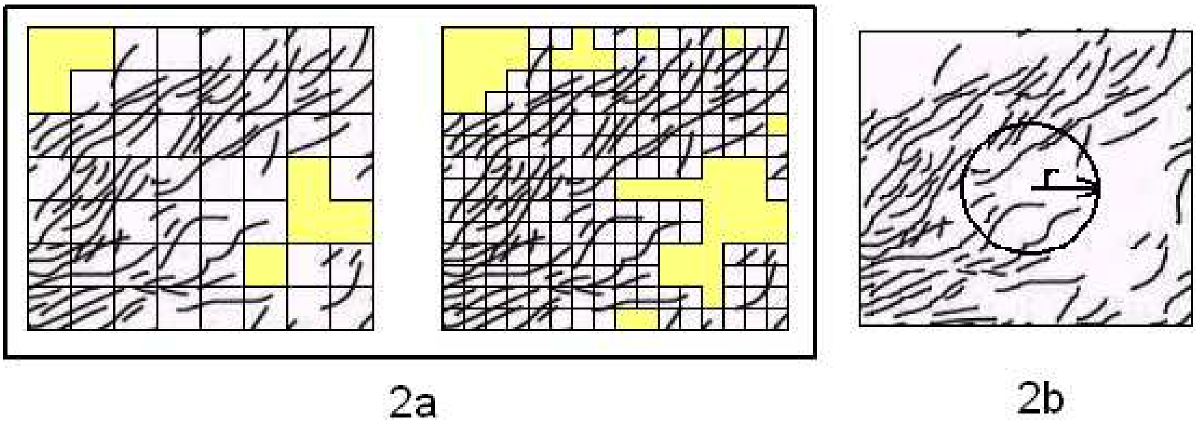

Figure 2: Classical methods used to calculate the fractal dimension. (2a) Shows the box-counting method. A system is coverded by a mesh of distinct size. Two different mesh sizes are shown. Empty boxes are shaded yellow. The number of boxes of size r required to cover all fractures is counted. It should vary with $N(r) = r^{-D}$, where D is the fractal dimension of the set. In a bilogarithmic plot D is the slope of the straight line. The mass method is shown in (2b). Here the fracture length L(r) included in circles of different radius r is measured. (redrawn from Bonnet et al., 2001)

getting a power-law distribution for faults, some geologists also observed exponential laws for the distribution of the fault size. Spyropoulos et. al (2002) suggested, that "these two populations are transition regimes between the end member stages".The two end member stages are an initial uncracked state and the saturated state, in which the cracks are evenly spaced avoiding the stress relaxation zone of its neighbour .

For this purpose they used an extension model with a brittle clay layer driven by an elastic bottom layer. They needed only a few parameters to analyse their experiments numerically.

If at any point the yield strength of the clay layer is exceeded, a crack will evolve with the opening h. They defined the strength of the material by using the slip weakening friction law. This is a huge simplification of the real process of material failure, but they had to do this to make the process computationally efficient. It also seems to capture the observations well. The authors reached a model of crack evolution by numerical modelling (Fig.3).

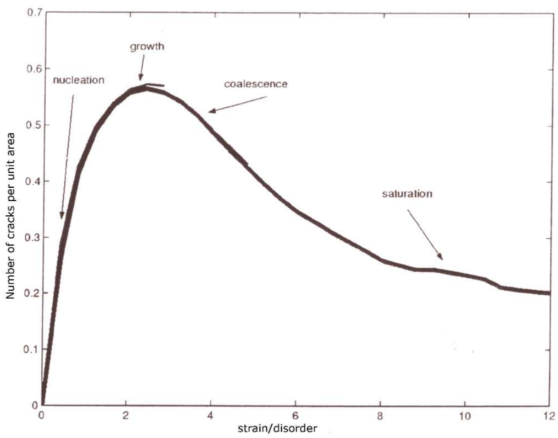

Figure 3: Number of active cracks per unit area evolving with increased strain. On the horizontal axis, excess strain is normalized by disorder g. Disorder $g = 10^{-4}, 10^{-3}, 10^{-2}$ with thicker lines corresponding to smaller g. The different strain regimes are labeled (modified from Spyropoulos et al., 2002).

Their model includes four different stages of evolution:

Nucleation At the beginning the number of cracks per unit area (fraction of active sites) increases with elastic strain.

Growth If the slip displacement becomes bigger, slip weakening starts to dominate the process and the slip localizes.

Coalescence In that stage the coalescence of cracks dominates the system.

Saturation Eventually the system reaches the stage of saturation, where cracks evolve, that are evenly spaced about one layer thickness apart, avoiding the stress relaxation zones of neighbouring fractures.

Varying different simulations could be made. Figure 4 shows the result of the simulation with 4 different values of slip weakening α.

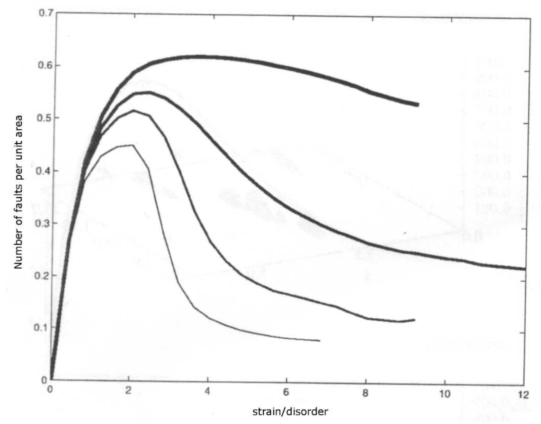

Figure 4: Number of active faults per unit area as a function of strain, for four different values of slip weakening . In all four cases there is a nucleation regime (the number of cracks increases with strain), the growth regime (the number of cracks peaks), a coalescence regime (the number of cracks starts to decrease) and a saturation regime (constant density of cracks spaced one layer thickness apart). As decreases the maximum number of cracks per unit area increases while it shifts towards larger strains. Thicker lines correspond to smaller α. ($\alpha = 0.1, 0.2, 0.3, 0.4$). Adapted from Spyropoulos et al., 2002.

The next step of the authors was what we are interested in: they examined the distribution of the crack sizes in the different regimes. The distribution of lengths varying with the strain (different lines) was plotted first in a log-linear-plot as shown in Figure 5. In such a plot, exponential laws should appear as straigth lines. For the very smallest loadings, this is true. Some of the distributions appear as a curved line and should be examined further. Plotted on a log-log-scale, these lines get straight, what is in accordance with a power-law-distribution (Fig.6).

The last step was to examine the largest cracks in the last regime. They are again plotted in an extended log-normal-plot and also show an exponential law (Fig. 7).

Spyropoulos et al. concluded, that "at very low strain the cracks are short, the crack population is dilute, there is very little interaction amongst the stress fields around them, and disorder dominates." If additional extension appears, the cracks start to propagate, "disorder and weakening compete" and the crack population reaches a power-law distribution. As the cracks grow longer, coalescence begins to dominate. This coalescence shows " a transition in the organization of the cracks to a regime in which the largest cracks have an exponential distribution." With still increasing strains the cracks grow and shadow zones evolve until the system reaches the maximum number of cracks allowed. This can be called saturation.

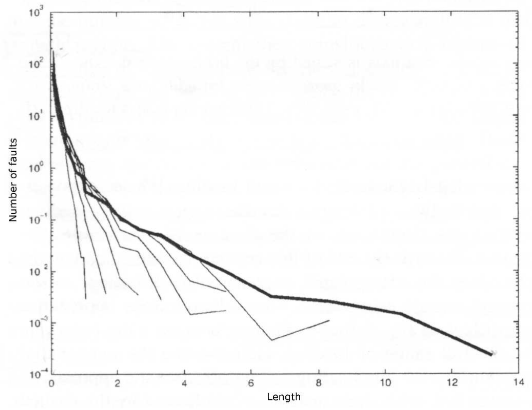

Figure 5: Log-linear plot of distribution of sizes for increased strain, at low strain values in the nucleation regime. Strain increases from left to right, with highest strain indicated by the bold line. Note the initially exponential distribution.(modified from Spyropoulos et al., 2002)

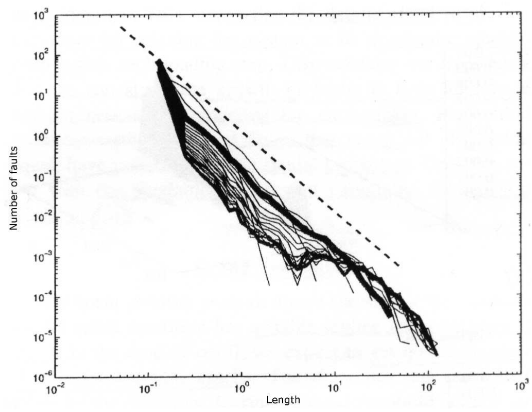

Figure 6: Log-log-plot of distribution of sizes for increased strain. Note the power-law distribution at intermediate strain, indicated by the thickest line, and the asymptotic distribution at higher strain, indicated by the other thick line. The dashed line shows a slope of -2 for comparison.(modified from Spyropoulos et al., 2002)

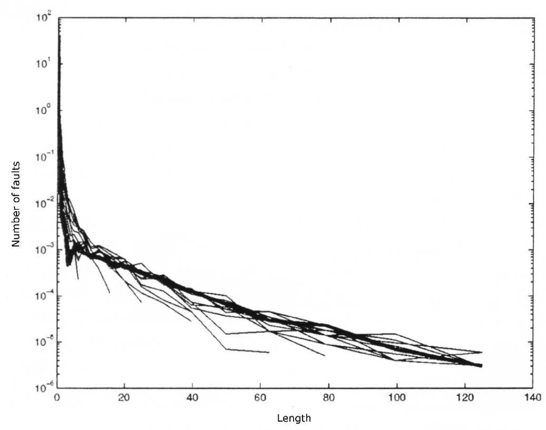

Figure 7: Log-linear plot of distribution of sizes for increased strain, at large strain values. Note the exponential distribution of largest events at large strains. The highest strain is highlighted with the thicker line (modified after Spyropoulos et al., 2002).

4.2 Comparison of experiment and nature

In this section the distributions found in experiments will be compared with the results of the analysis of different measured fault sets. Many authors attempted to find scaling laws for natural faults. It is obvious that the scientists, who used data from only one set of fractures reached distributions very similar to the results of *Spyropoulos*(2002). The best agreement seems to occur with the results of Gupta and Scholz (2000), who examined faults in the Afar depression. They observed a strain regime transition from a power law to an exponential law for the frequency-size distribution (Figure 8), when faults reach a certain density. For the faults of the Afar depression this density equals 0.6 km of fault length per square kilometer. This corresponds with a strain of 6-8%. Above this density faults grow mostly by coalescence until the area is saturated because faults become pinned by the stress field of nearby faults.

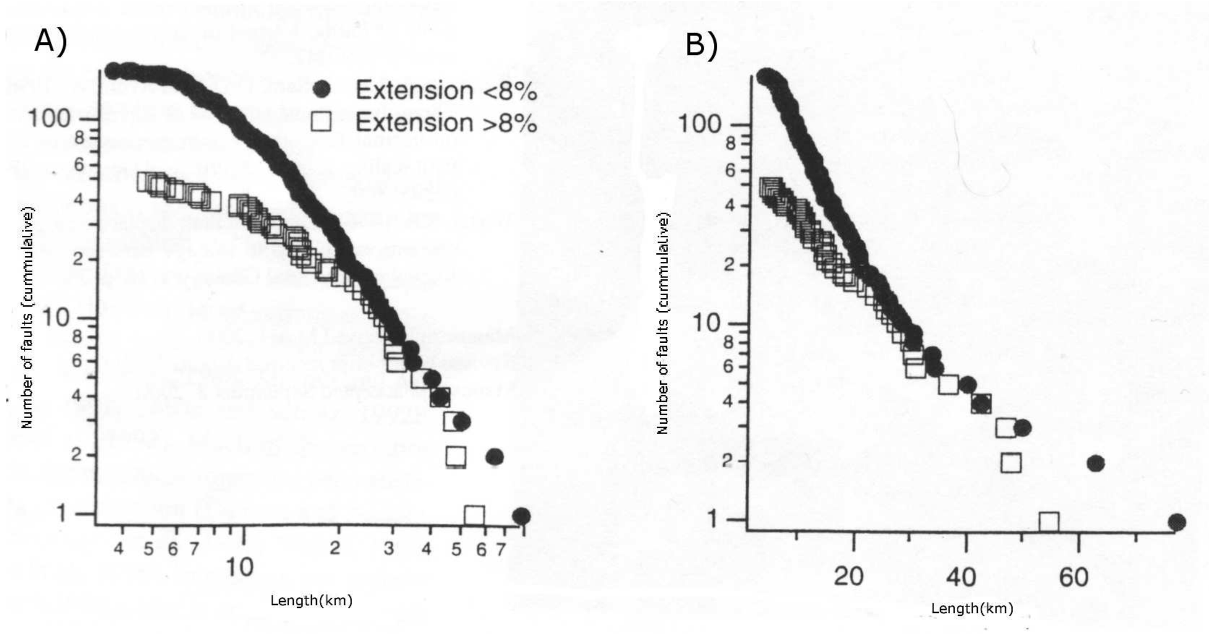

Figure 8: A: Cumulative number of faults vs. fault length, plotted on log-log-axes. B: data plotted on log-linear axes (modified from GUPTA and SCHOLZ, 2000).

Another field of research are the scaling relationships between displacement and length. As said before the length and the displacement should show the same proportions to each other, ignoring their order of magnitude. That means we would expext a linear relationship.

One of the first scientists who searched for these relationships was WATTERSON (1986). He examined data from different geological settings and draw the conclusion, that the relationship between the total displacement and the fault width can not be linear. He found a model with a relationship $D = c * L^2$ with D is the total displacement, L is the length of the fault and c is a constant. He assumed, that the succesive slip events increase by a constant increment k. In Figure 9 the lines F, F' and F" represent growth curves for this model with varying k-values. D depends on k with $D = u^2/2k$ (u is the last slip).

Different fault sets should not be analysed together, because varying material properties will have an influence on the scaling relations. It may be possible that the single sets show other displacement-length scaling than all the sets together. Authors who examined this phenomenon are for example COWIE AND Scholz (1992) or DAWERS et al. (1993).

COWIE and SCHOLZ (1992) used data from very different individual data sets from other authors. They recognized every single data set to fit linaer with different constants of proportionality (Figure 10). This reflects different material properties. They critizised other authors for taking all data together and interpret this as one set of faults. Because of the different material properties, they must find laws differing from the distributions of single sets of faults. One explanation for this observation is, that large faults often rupture stronger rocks.COWIE and SCHOLZ(1992) concluded that "data that span a much greater scale range for faults in a single tectonic environment and rock type" are required.

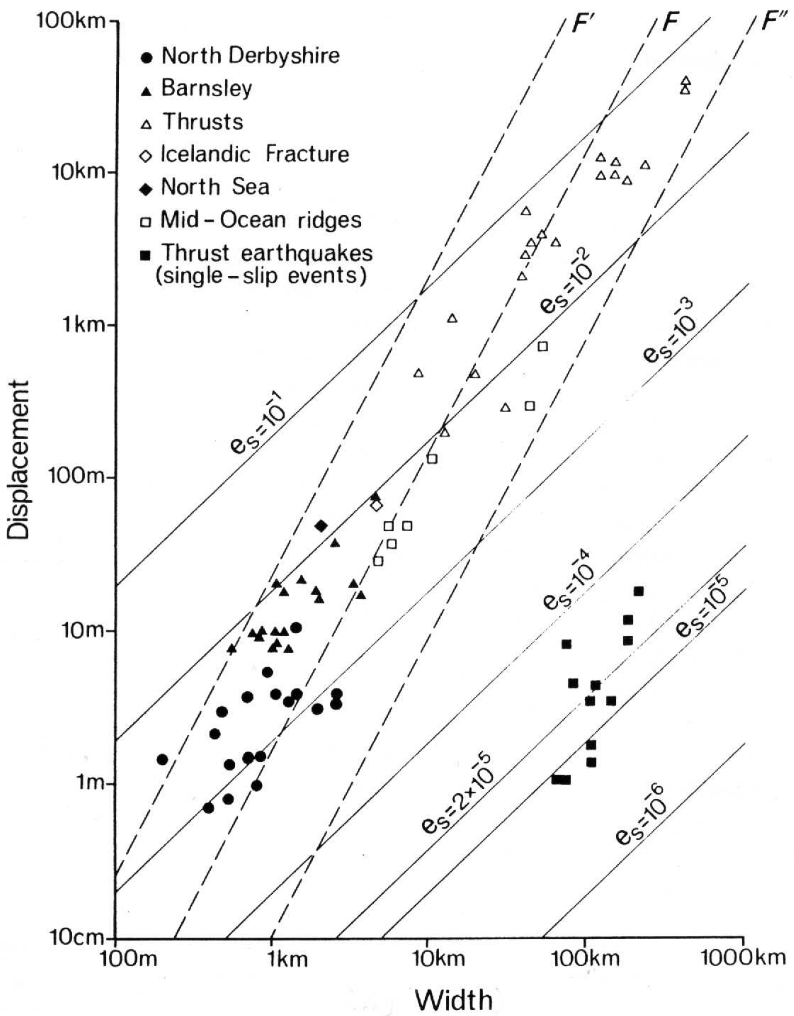

Figure 9: Width versus maximum displacement for faults, thrusts and groups of faults and slip versus width for seismic events; e_s is linear strain required for compatibility and F, F' and F" are model growth curves (modified from Watterson, 1986).

5 Sampling effects and other problems

The biggest problem in analysing the scaling laws of faults is that for a reliable distribution law one would need a range of data of more than two orders of magnitude. To reach that, many authors used data from different geological and tectonic settings, the distribution- and scaling laws of which may depend on different material properties. As mentioned before for a good analysis a data set over several orders of magnitude but in a single tectonic environment and rock type is necessary.

Other problems arise in the finite size of the sampled domain and in the resolution of the technique. These effects are called truncation and censoring after Bonnet et al. (2001).

Truncation is the underestimation of small fractures due to resolution limitations. These effects can be identified in the density distribution. The slope then goes through zero and becomes positive for the smallest fractures. *Censoring* occurs because of the finite size of the area examined. The largest faults tend to be incompletely observed.

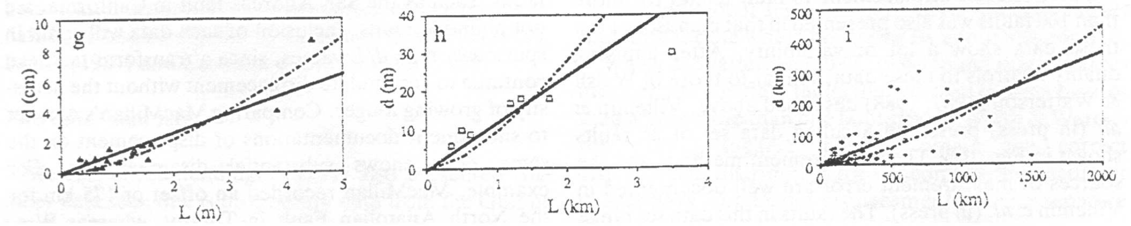

Figure 10: Plots of maximum fault displacement for three different data sets: (g)Muraoka and Kamata(1983), (h) Opheim and Gudmundsson(1989) (i) MacMillan (1975), The solid line represents a linear fit. The non-linear fits tend to overestimate the displacement for the larger faults (modified from COWIE and SCHOLZ, 1992).

They are cut by the borders of the observed area. Another cause for the same effect is the subjective choice of the sample region. Often large faults are automatically excluded. The two effects are easily observed in Figure 8. It is not possible to observe a set of several hundreds of faults in 3D. So 2D-cuts through a 3D-system are used for getting enough data for a statistical examination. It is not yet known if this cut effect has an influence on the distribution found at the surface. To learn something about that, it would be necessary to gather a data set with the three dimensional shape of the fractures.

6 Discusssion and conclusion

Searching for a fractal distribution of faults, we would like to find two different scaling relationships. Faults should show a power-law size distribution and a linear scaling of the strain with the length of the fault. If we have a closer look at the long known laws for earthquakes, they seem to show a fractal behaviour. On the one hand they have a power-law distribution as mentioned before. On the other hand earthquakes also show a linear relationship between slip increment and rupture length.

Maybe one would expect faults to be distributed the same way, because they are often closely connected with earthquakes. According to the model of SPYROPOULUS et al. the power-law distribution appears only in the process of the undisturbed growth. In the processes of nucleation and of coalescence and saturation they observed an exponential law.

SCHOLZ et al. (1993) found a linear scaling relationship between strain and length of faults, but they limited their model on faults that grow as isolated cracks. They said that if coalescence, segmentation or even saturation occurs, the scaling law can not be applicated. A striking example is if two faults coalesce, the new fault has the same throw, but its length is the sum of the initial faults. Maybe the fault now continues to accumulate strain without growing in length, until the appropriate D/L ratio is reached. As long as this is not reached, the fault could never fit a linear scaling law.

Scientists found a mathematical model that can be applied on a part of the evolution of faults.

But in natural systems we find things like layered materials with different strength or two sets of faults that cut each other. It is difficult to get a useful data set.

"Useful" means in this sense that the faults must have evolved in homogeneous material, they should cover several orders of magnitude and they must evolve independend from one another without disturbing the growth of neighbouring faults. This can explain why the fracal model fits best with the experimental data. Those data can never reach the complexity of the nature.

But after all the model of fractals fits relatively good in many cases and so it could help to understand the complex evolution of faults.

7 References

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